OPTIMIZATION OF HEAT TRANSFER PROCESSES IN ENCLOSING STRUCTURES OF ARCHITECTURAL MONUMENTS LOCATED OUTSIDE URBAN AGGLOMERATION

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Abstract. Numerous architectural monuments throughout Ukraine were once proud and were the center of historical events. With a glorious and rich history, but forgotten by people because they are located outside the city limits and are not included in well-known tourist routes. Historic buildings are examples of neoclassicism, baroque, neo-gothic and many other styles. During Soviet times, they were turned into barracks, chemical storage facilities, food and weapon warehouses, and now they are empty and collapsing. A correct and calculated approach to reconstruction, restoration and monitoring of their further condition will make it possible to turn such objects into the cultural and historical center of a rural agglomeration. This will help maintain historical memory, preserve culture and heritage, and also give economic impetus to the development of the area where they are located. Leakage of worn-out enclosing structures is the biggest problem of abandoned architectural monuments. This causes uneven wetting of the structures. And then the task is posed of how to properly dry such enclosing structures and not damage them. All these old building envelopes are colloidal capillary porous bodies (CCPB). Drying of such massive structures occurs due to heating. The problem of the optimal heating rate of a massive colloidal capillary-porous body is considered. The restrictions that are imposed on the internal thermal stresses in the specified body are taken into account, especially at the first low-temperature stage when heating the colloidal capillary porous bodies. Due to the temperature distribution over the mass (over the cross-section) of the colloidal capillary porous bodies, compressive and tensile stresses arise inside the body. Colloidal capillary porous bodies can be destroyed by these stresses, leading to the appearance of various microdefects.

Keywords: heat transfer; capillary-porous; heating; thermal tension; architectural monuments; preservation.

Introduction

Throughout Ukraine, there are many architectural monuments that were once proud and which were the epicenter of historical events. With a glorious and rich history, but forgotten by people because they are located outside the city limits and are not included in well-known tourist routes. Historic buildings are examples of neoclassicism, baroque, neo-gothic and many other styles. During Soviet times, they were turned into barracks, chemical storage facilities, food and weapon warehouses, and now these buildings are empty and collapsing. A correct and scientifically based approach to reconstruction, restoration and monitoring of their further condition will make it possible to turn such objects into the cultural and historical center of a rural agglomeration. This will help maintain historical memory, preserve cultural heritage, and also give an economic boost to the development of the area where they are located. Leaks in worn-out building envelopes are the biggest problem facing abandoned architectural monuments. This causes uneven wetting of the structures. And then the task is posed of how to properly dry such enclosing structures and not damage them. All these old building envelopes are colloidal capillary porous bodies (CCPB). Drying of such massive structures occurs due to heating. As is known from [1-15], many researchers have studied the optimal speed of heating of massive bodies (including colloidal capillary-porous bodies – CCPB).

When such bodies are heated, thermal stress arises, which is often the main reason for the limitation that limits the heating rate. Especially at the first low-temperature stage, when heating CCPB, its outer layers have a temperature above the average by weight (over the cross section), and the temperature of the inner layers is significantly lower than the average temperature. Due to this temperature distribution, the outer layers of CCPB tend to expand. This expansion is prevented by the inner layers of the body, since due to the lower temperature they expand less, so the outer layers of CCPB experience compression, and the inner layers experience tension. In accordance with this, compressive and tensile stresses arise inside the body. These stresses can lead to the destruction of CCPB and the appearance of various microdefects.

Optimum conditions for the speed of heating of a massive CCPB are taking into account the limitations on thermal stress and setting the problem of optimal control. The problem of heating an infinite plate (CKPT) with a thickness of $2\cdot s$, m is considered. We denote the temperature $\tilde{u}(\tau)$, $0 \leq \tau \leq \tau_0$

dependent on time τ, s. This function will be a controlling influence. Of practical interest is the case of limited ambient temperature. Therefore, we set the following restrictions for $\tilde{u}(\tau)$.

$$A_1 \leq u(\tau) \leq A_2, 0 \leq \tau \leq \tau_0, \quad (1)$$

where A_2 and A_1 – maximum and minimum possible heating temperatures, respectively.

Next, we will assume that at the initial moment of time the temperature at all points of the plate is the same and equal $t_0(x,0) = T_0 = const$ at the coordinate x, m.

The temperature distribution in the body, which must be obtained as a result of heating, is considered to be equal to some constant temperature $t_3(x) = const$, i.e. the task is to change the temperature distribution inside the body from the constant T_0 to the constant C.

We set the origin of coordinates in the center of the plate, since the heat exchange between the environment and each of the surfaces occurs in the same way (symmetric problem). With $-s \leq x \leq s$, $\tau > 0$; $u(\tau)$ is a piecewise continuous function of time satisfying condition (1).

Temperatures A_2, A_1, T_0, C satisfy the following obvious inequalities: $A_2 > A_1$, $C > T_0$. The latter means that the problem of heating CCPB will be considered in the future, but the obtained results can easily be used to solve the problem of cooling as well. In addition, we consider the following inequality to be satisfied:

$$A_1 < C < A_2, \quad (2)$$

that is, the value of the temperature C to which the CCPB should be heated does not exceed the range of the ambient temperature change.

Materials and methods

We will introduce the following dimensionless coordinates adopted in heat engineering: $\varphi = a\tau/s^2$ dimensionless time (Fourier criterion – Fo, in the future we will call φ simply time); $a = \lambda/(c_p \cdot \rho)$ –thermal diffusivity, $m^2 \cdot s^{-1}$; λ – thermal conductivity coefficient, $W \cdot m^{-1} \cdot K^{-1}$; c_p – specific heat capacity, $J \cdot kg^{-1} \cdot K^{-1}$; ρ – density, $kg \cdot m^{-3}$; $l = x/s$ – dimensionless thickness, $-1 \leq l \leq +1$; $b = (\alpha/\lambda) \cdot s$ – Bio criterion (Bi); α – heat transfer coefficient, $W \cdot m^{-2} \cdot K^{-1}$; $\nu = 2(C - T_0)/(A_2 - A_1)$ – dimensionless temperature (initial condition criterion); $æ = (A_2 + A_1 - 2C)/(A_2 - A_1)$ dimensionless temperature (criterion of heating asymmetry).

It follows from (2), that $|æ| < 1$. We denote the new dimensionless temperature as $Q(l, \varphi) = 2[u\text{-}C]/(A_2 - A_1)$.

Let us consider the process of heating a one-dimensional plate (model of an array of CCPB), the temperature distribution inside which is described by the thermal conductivity equation.

$$\frac{\partial Q(l,\varphi)}{\partial \varphi} = \frac{\partial^2 Q(l,\varphi)}{\partial l^2} \quad (3)$$

with initial and boundary conditions:

$$Q(l, 0) = -\nu, \quad (4)$$

$$\left.\frac{\partial Q(l,\varphi)}{\partial l}\right|_{l=1} = b \cdot [u(\varphi) - Q(1, \varphi)], \quad (5)$$

$$-\left.\frac{\partial Q(l,\varphi)}{\partial l}\right|_{l=1} = b \cdot [u(\varphi) - Q(-1, \varphi)], \quad (6)$$

The main effect must also satisfy the condition:

$$-(1 - æ) \leq u(\varphi) \leq 1 + æ. \quad (7)$$

The thermal stress distribution in the plate is described by the relation [16]:

$$\sigma(l, \varphi) = \frac{\beta \cdot E}{(1-\Theta)} \cdot \left[\overline{Q}(\varphi) - Q(l, \varphi)\right], \quad (8)$$

where σ – stress in CCPB, $N \cdot m^{-2}$;
β – coefficient of linear temperature expansion of the body material, K^{-1};
E – modulus of elasticity, Pa;
Θ – Poisson's ratio;

$\overline{Q}(\varphi)$ – average temperature of the body across the mass section, K.

The diagram of the thermal stress distribution during instantaneous measurement of the temperature of the heating medium is shown in Fig. 1. It shows that the greatest tensile stress occurs in the middle of the plate at $l = 0$.

Since the most dangerous tensile stresses during heating (they can destroy a massive CCPB by forming microcracks in it), the following limitation is obtained:

$$\sigma(l,0) = \frac{\beta \cdot E}{(1-\Theta)} \cdot \left[\overline{Q}(\varphi) - Q(0,\varphi)\right] \leq \sigma^* , \quad (9)$$

where σ^* – maximum allowable tensile stress.

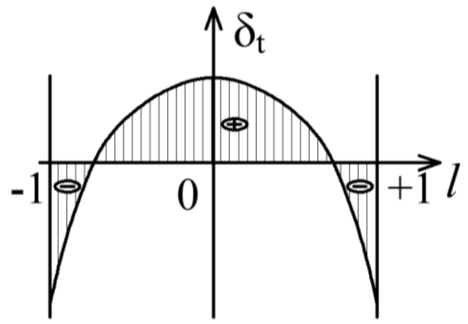

Fig. 1. **Circuit diagram of distribution of thermal stress with instantaneous change in the temperature of the heating medium**

The last condition (9) can be rewritten in dimensionless form for the temperature stress criterion S:

$$S = \frac{\sigma(0,\varphi)}{\sigma^*} = \gamma \cdot \left[\overline{Q}(\varphi) - Q(0,\varphi)\right] \leq 1, \quad (10)$$

where $\gamma = \frac{\beta \cdot E}{\sigma^* \cdot (1-\Theta)}$ – dimensionless parameter.

Thus, the task of optimal management is to find such a management that, when fulfilling ratios (3-7), zero distribution is achieved in the minimum time:

$$Q(l,\varphi_0) = 0, \text{ for all } -1 \leq l \leq +1. \quad (11)$$

As shown in [15], the task of optimal control in this case can be reduced to an infinite system of equations:

$$x_k(\varphi_0) = 0, k = 1,2,3 \dots , \quad (12)$$

where

$$x_k(\varphi_0) = -\nu \cdot \exp\left(-\mu_k^2 \cdot \varphi\right) + \int_0^{\varphi} u(\zeta) \cdot \mu_k^2 \cdot \exp\left(-\mu_k^2 \cdot [\varphi - \zeta]\right) d\zeta, \quad (13)$$

where μ_k – different real positive solutions of the characteristic equation, $(1/b) \cdot \mu = \text{ctg } \mu$, $0 < \mu_1 < \mu_2 < \mu_3 \dots < \mu_k < \dots$

ζ – dimensionless variable of integration.

Differentiating the last equality (13) with respect to φ, we obtain an infinite system of differential equations:

$$x_k(\varphi) = -\mu_k^2 \cdot (\varphi) + \mu_k^2 \cdot u(\varphi) , k = 1,2,3 \dots , \quad (14)$$

with initial conditions:

$$x_k (0) = -\nu , k = 1,2,3 \dots . \quad (15)$$

Results and discussion

Let us now express the temperature stress criterion $S(\varphi)$ in terms of coordinates $x_k (\varphi)$ of system (14), using formula (13). We will get:

$$S(\varphi) = \gamma \cdot \sum_{n=1}^{\infty} B_n \cdot \left[-\nu \cdot \exp\left(-\mu_n^2 \cdot \varphi\right) + \mu_n^2 \cdot \int_0^{\varphi} u(\zeta) \cdot \mu_n^2 \cdot \exp\left[-\mu_n^2 \cdot (\varphi - \zeta)\right] d\zeta\right], \quad (16)$$

where

$$B_n = A_n \left(\frac{\sin\mu_n}{\mu_n} - 1 \right). \tag{17}$$

Accordingly, constraint (10) can be rewritten in the form:

$$\gamma \sum_{n=1}^{\infty} B_n \cdot X_n(\varphi) \leq 1. \tag{18}$$

Thus, the task of optimal heating control of CCPB, taking into account the limitations on thermal stress, is formulated as follows. Find such a control $u(\varphi)$ that, subject to the constraints (7) and (18), the system (14) is passed from the initial state (15) to the final state (12) in the minimum time $\varphi = \varphi_0$.

This means that the task under consideration is reduced to the optimal control of an infinite system of differential equations (equations in the Banach space [15]) when limited to a linear combination of phase coordinates (18).

It is obvious that with the given restriction (18) and the restriction on the control influence $u(t)$ it is possible to choose a value of the initial temperature distribution ν so large in terms of modulus that with any admissible control the stress in CCPB will exceed the admissible limit.

To find this limiting relationship between the value of ν and the value of b, we substitute $u(\varphi) = æ - 1$ in (14) and integrate this equation. We substitute the obtained coordinate value into the expression for $S(\varphi)$. Then we will have:

$$\gamma(-\nu + æ - 1) \cdot \sum_{n=1}^{\infty} B_n \cdot \exp(-\mu_n^2 \cdot \varphi) \leq 1.$$

Where we will get:

$$P = -\nu + æ - 1 \leq \frac{1}{\gamma \sum_{n=1}^{\infty} B_n \cdot \exp(-\mu_n^2 \cdot \varphi)}. \tag{19}$$

Fig. 2 shows the graph of the dependence of the maximum permissible value of the absolute value of $P_{\min}$ on the Bio criterion (b) at $\gamma = 1$ and $æ = 0$ in logarithmic scale. If the point is located above the line, the system cannot be transferred to the required state without exceeding the thermal stress $\sigma(0, \varphi)$ of the permissible value σ^*.

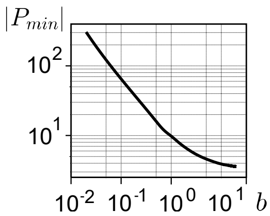

Fig. 2. **Circuit diagram of distribution of thermal stress with instantaneous change in the temperature of the heating medium**

In work [15] it is shown that a good approximation of the problem of optimal heating of a massive body is obtained if we consider the "truncated" system of equations (14) with a small number of equations $k = 2,3,4$. As it was shown, this is explained by the fact that the exponent (or μ_k^2) inverse of the "time constant" in equations (14) grows very quickly in modulus, because:

$$\mu_k > \frac{(2k-1)\pi}{2}, k = 1,2,3 \ldots$$

Here we will limit ourselves to the system of two equations ($k = 2$):

$$\begin{cases} x_1(\varphi) = -\mu_1^2 \cdot x_1 \cdot (\varphi) + \mu_1^2 \cdot u(\varphi), x_1(\varphi) = \frac{dx_1(\varphi)}{d\varphi} \\ x_2(\varphi) = -\mu_2^2 \cdot x_2 \cdot (\varphi) + \mu_2^2 \cdot u(\varphi), x_2(\varphi) = \frac{dx_2(\varphi)}{d\varphi}. \end{cases} \tag{20}$$

With initial and final conditions:

$$x_1(0) = x_2(0) = -\nu, x_1(\varphi_0) = x_2(\varphi_0) = 0. \tag{21}$$

Let us assume that:

$$\gamma \cdot \left[B_1 \cdot x_1(\varphi) + B_2 \cdot x_2(\varphi) \right] \leq 1, B_1 < 0 \text{ and } B_2 > 0. \tag{22}$$

If we denote $-\gamma B_1 = C_1$ and $\gamma B_2 = C_2$ then $C_1 > 0$ *and* $C_2 > 0$, inequality (22) will take the form:

$$-C_1 \cdot x_1(\varphi) + C_2 x_2(\varphi) \leq 1, 0 \leq \varphi \leq \varphi_0.$$

When solving our problem, we will limit ourselves to considering the case when $\nu \geq 0$. As can be seen from the system of equations (20)-(22), if $\nu < 0$, then it is necessary to find the optimal control $u(\varphi)$ corresponding to the initial conditions $x_1(0) = x_2(0) = -\nu$. Then the optimal control will be equal to $\{-u(\varphi)\}$.

Thus, on the phase plane (x_1, x_2) the task is reduced to constructing the trajectory of the characteristic point of the system (x_1, x_2). The trajectory starts on the bisector of the third quadrant at the point $(-\nu, -\nu)$ and ends at the origin of coordinates $(0, 0)$. Moving along this trajectory, this point must reach the origin of coordinates in the minimum time, without entering the forbidden area lying above the straight line:

$$-C_1 \cdot x_1 + C_2 \cdot x_2 = 1. \tag{23}$$

For sufficiently small $|\nu|$ the optimal trajectory will not reach the constraint. Therefore, due to the fact that this system is a second-order linear system with real eigenvalues $(-\mu_1^2)$ and $(-\mu_2^2)$ the optimal control $u(\varphi)$ will have one exception. The switching line in this case is easily calculated by "reverse time" $\varphi \leq 0$ [11]. The parametric control of the switching line is determined by integrating the equations of motion (20) at $u(\varphi) = -1$ in the negative direction of time $\varphi \leq 0$ with the initial conditions $x_1(0) = x_2(0) = 0$. These equations have the form:

$$\begin{cases} x_1(\varphi) = \mu_1^2 \cdot x_1 \cdot (\varphi) + \mu_1^2 \\ x_2(\varphi) = \mu_2^2 \cdot x_2 \cdot (\varphi) + \mu_2^2 \end{cases}. \tag{24}$$

Integrating these equations, we have:

$$\begin{cases} x_1(\varphi) = 1 - exp\ (\mu_1^2 \cdot \varphi) \\ x_2(\varphi) = 1 - exp\ (\mu_2^2 \cdot \varphi) \end{cases}. \tag{25}$$

Excluding the parameter φ from the last system, we obtain the equation of the switching line (Fig. 3):

$$x_2 = 1 - (1 - x_1)^{\left(\frac{\mu_2}{\mu_1}\right)^2} = f(x_1). \tag{26}$$

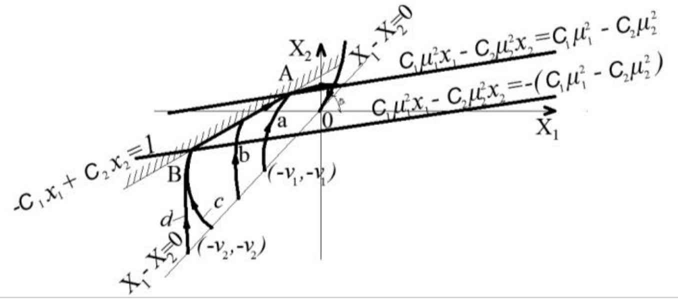

Fig. 3. **Optimal trajectory of the characteristic point of the system on the phase plane (x_1, x_2):** lines a, b, c, d are variants of the possible trajectory of the characteristic point, at different initial coordinates

As ν increases, a moment will come when the optimal trajectory will touch the limit line. Let us calculate the coordinates of this point of contact. By integrating the system (20), we first determine the equation of the optimal trajectory $u = 1$ under the initial conditions $x_1(0) = x_2(0) = -\nu$.

$$\begin{cases} x_1(\varphi) = (1 - \nu) \cdot exp\ (-\mu_1^2 \cdot \varphi) + 1 \\ x_2(\varphi) = (1 - \nu) \cdot exp\ (-\mu_2^2 \cdot \varphi) + 1 \end{cases}. \tag{27}$$

Excluding φ from (27), we obtain the required equation:

$$x_2 = (1 - \nu) \cdot \left(\frac{x_1 - 1}{1 - \nu}\right)^{\left(\frac{\mu_2}{\mu_1}\right)^2} + 1 = f_2(x_1). \tag{28}$$

The coordinate x_1 of the desired point of contact A (Fig. 3) and the maximum value of the parameter $\nu = \nu_1$, at which the optimal trajectory will touch the limit line for the first time, can be found from the system of equations:

$$\frac{\delta x_2}{\delta x_1} = \left(\frac{\mu_2}{\mu_1}\right)^2 \cdot \left(\frac{x_1-1}{1-\nu}\right)^{\left(\frac{\mu_2^2}{\mu_1^2}-1\right)} = \frac{C_1}{C_2}. \qquad (29), (30)$$

$$-C_1 \cdot x_1 + C_2(1-\nu) \cdot \left(\frac{x_1-1}{1-\nu}\right)^{\left(\frac{\mu_2^2}{\mu_1^2}\right)} + C_2 = 1$$

From the first equation (29), we determine:

$$\left(\frac{x_1-1}{1-\nu}\right)^{\left(\frac{\mu_2^2}{\mu_1^2}\right)} = \frac{C_1}{C_2} \cdot \left(\frac{\mu_1}{\mu_2}\right)^2 \cdot \frac{x_1-1}{1-\nu}. \qquad (31)$$

Let us substitute the specific expression in (30). Then we get:

$$-C_1 \cdot x_1 + C_1 \cdot \left(\frac{\mu_1}{\mu_2}\right)^2 \cdot (x_1 - 1) + C_2 = 1. \qquad (32)$$

From here:

$$x_1 = \frac{1-C_2 + C_1 \cdot \left(\frac{\mu_1}{\mu_2}\right)^2}{C_1 \cdot \left(\frac{\mu_1}{\mu_2}\right)^2 - C_1}. \qquad (33)$$

Using equation (31), it is easy to find:

$$\nu_1 = \left[\frac{C_2}{C_1} \cdot \left(\frac{\mu_2}{\mu_1}\right)^2\right]^{\left(\frac{\mu_1^2}{\mu_2^2-\mu_1^2}\right)} \cdot \frac{[1-C_2 + C_1]}{C_1 \cdot \left(\frac{\mu_1}{\mu_2}\right)^2 - C_2}. \qquad (34)$$

Thus, with $\nu < \nu_1$, the optimal trajectory does not reach the limit line.

When $\nu = \nu_1$ the optimal trajectory (line a, Fig. 3) only touches the constraints. The optimal process in this case is shown in Fig. 4a.

When $\nu > \nu_1$ the characteristic point "rests" against the restriction, and in order not to go beyond the permissible area, it must move along the restriction line to point A (line b, Fig. 3). The conditions under which the characteristic point can move along the straight line (23) have the form:

$$\frac{x_2(\varphi)}{x_1(\varphi)} = \frac{C_1}{C_2} = \frac{\mu_2^2 \cdot x_2(\varphi) - \mu_2^2 \cdot u}{\mu_1^2 \cdot x_1(\varphi) - \mu_1^2 \cdot u}. \qquad (35)$$

From here

$$u(\varphi) = \frac{1}{\frac{C_1}{C_2} \cdot \mu_1^2 - \mu_2^2} \cdot \left(\frac{C_1}{C_2} \cdot \mu_1^2 \cdot x_1(\varphi) - \mu_2^2 \cdot x_2(\varphi)\right), \qquad (36)$$

or

$$u(\varphi) = \frac{C_1^1 \cdot \mu_1^2 \cdot x_1(\varphi) - C_2^1 \cdot \mu_2^2 \cdot x_2(\varphi)}{[C_1 \cdot \mu_1^2 - C_2 \cdot \mu_2^2]}. \qquad (37)$$

In order to find a clear dependence of control $u(\varphi)$ on time in a given section, the last relation for $u(\varphi)$ must be substituted into the initial system of equations (20). These equations are linear equations and their solutions are easy to find given the appropriate initial conditions. Having determined the explicit dependences of $x_1(\varphi)$ and $x_2(\varphi)$ from these equations and substituting them into equation (37), we will find the explicit dependence of $u(\varphi)$. Intermediate calculations are not given in the work.

The control $u(\varphi)$ determined by formula (37) must be subject to the main constraint (7). On the segment BA, the characteristic point can be kept on the limit line without entering the forbidden area with the help of admissible control. Allowable for movement, the segment BA of the straight limit is determined by the intersection of the limit line itself with two straight lines $C_1 \cdot \mu_1^2 \cdot x_1 - C_2 \cdot \mu_2^2 \cdot x_2 = C_1 \cdot \mu_1^2 - C_2 \cdot \mu_2^2$; $C_1 \cdot \mu_1^2 \cdot x_1 - C_2 \cdot \mu_2^2 \cdot x_2 = -(C_1 \cdot \mu_1^2 - C_2 \cdot \mu_2^2)$ (Fig. 3).

Point B corresponds to the minimum value $\nu_{min} = \nu_2$, determined from the graph in Fig. 3, in which the characteristic point does not go beyond the permissible area.

Thus, the ray corresponding to the bisector of the third quadrant is divided by the points $(-\nu_2, -\nu_2), (-\nu_1, -\nu_1)$ into three parts.

If the initial point $(x_1(0), x_2(0) = (-\nu, -\nu)$ lies further from the origin of coordinates than the point $(-\nu_2, -\nu_2)$, then there is no optimal control.

If the initial point coincides with the point $(-\nu_2, -\nu_2)$ (line d, Fig. 3), then the optimal process has the form shown in Fig. 4 d.

If the initial point lies between the points $(-\nu_2, -\nu_2)$ and $(-\nu_1, -\nu_1)$ then the optimal process has the form shown in Fig. 4 b, c.

Fig. 4a shows the type of optimal control when the stress limit is not reached.

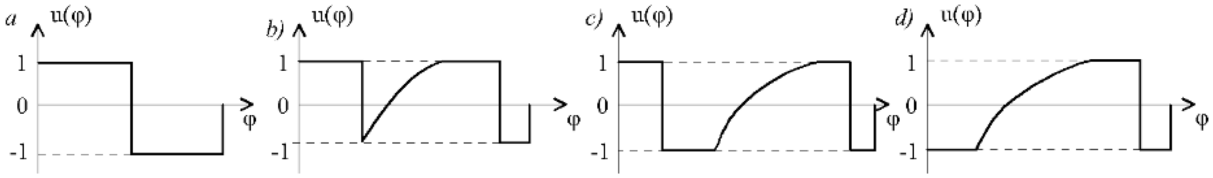

Fig. 4. **Appearance of optimal control $u\ (\varphi)$for different initial conditions $(x_1(0), x_2(0))$:**
a) $(-\nu_1, -\nu_1)$; b) $(-\nu_1', -\nu_1')$; c) $(-\nu_1'', -\nu_1'')$; d) $(-\nu_2, -\nu_2)$; $|\nu_2| > |\nu_1''| > |\nu_1'| > |\nu_1|$

Conclusions

A solution to the problem of optimal heating rate of a massive CCPB has been obtained. In accordance with Fig. 2, the dependence of the modulus $P_{\min}$ on the Bio criterion (b) was established. This dependence has the character of a monotonically decreasing function, i.e., when increasing the parameter Bio from 0,02 to 20, the value of the modulus $P_{\min}$ falls from 305.7 to 3.57. The curve $P_{\min}$(b) itself serves as a boundary that divides the whole plane of parameters $P_{\min}$ b into upper and lower parts relative to the curve. If the parameters of the system (medium, material) lie in the upper region (above the curve), then it cannot be transferred to the required state without applying an appropriate external thermal stress exceeding some threshold value. If the parameters of the system lie in the lower region (below the curve), then it is not necessary to apply additional thermal stress from outside. Without any compulsion, the system itself will move to the required state. The restrictions that are imposed on the internal thermal stresses in the specified body were taken into account, especially at the first low-temperature stage of heating. This means that the task is reduced to the optimal control of an infinite system of differential equations when limited to a linear combination of phase coordinates. Taking into account the given restrictions, it is possible to choose such a value (modulo) of the initial temperature distribution ν and determine the optimal heating control trajectory. For sufficiently small $|\nu|$ the optimal control trajectory $u\ (\varphi)$ does not reach the limits imposed on the thermal stress that occurs in a massive CCPB and has a single switching ($|\nu| < |\nu_1|$). When $|\nu| = |\nu_1|$, the optimal trajectory just touches the constraint. Optimal control $u\ (\varphi)$ corresponds to Fig. 4a and has one switch. When $|\nu| > |\nu_1|$, the characteristic point "rests" on the restriction, and in order not to go beyond the permissible area, it must move along the straight line of the restriction and point A (Fig. 3). Optimal control $u\ (\varphi)$ corresponds to Fig. 4 b, c and has three switching points. When $|\nu| = |\nu_2|$, the characteristic point does not go beyond the permissible area. At the same time $|\nu_1| = \min, |\nu_2| = \max$. Optimal control $u\ (\varphi)$ corresponds to Fig. 4 d and has two switching points. When $|\nu| > |\nu_2|$ there is no optimal control $u\ (\varphi)$ taking into account the restrictions imposed on the thermal stress in CCPB.

Author contributions

Conceptualization, Y.C.; methodology, A.M. and M.S.; software, Y.C.; validation, S.R. and O.M.; formal analysis, Y.C., A.M. and M.S.; investigation, Y.C., A.M., M.S., S.R. and O.M.; data curation, A.A., V.B. an J.I.; writing – original draft preparation, Y.C.; writing – review and editing, A.M. and M.S.; visualization, S.R. and O.M.; project administration, A.M.; funding acquisition, S.R. All authors have read and agreed to the published version of the manuscript.

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